Sydney Girls High School

2002
TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION

Mathematics

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2002 HSC Examination Paper in this subject.

Candidate Number

General Instructions

- Reading Time - 5 mins
- Working time - 3 hours
- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied
- Board-approved calculators may be used.
- Diagrams are not to scale
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

Question 1.

(a) Solve for x

$$(2x-5)(x+2)=0$$

(b) If $x=4$, evaluate $\dfrac{e^x-1}{e^x+1}$ correct to 3 significant figures

(c) Solve for x

$$|\, x+5 \,| = 3$$

(d) Simplify $\dfrac{3x}{5}-\dfrac{x-1}{2}$

(e) Express $0.\dot{3}\dot{1}$ as a fraction in lowest terms.

Question 2.

A $(2,3)$ B $(-2,-7)$

(a) Find
- (i) length of AB
- (ii) midpt of AB
- (iii) gradient of AB
- (iv) equation of AB
- (v) y – intercept of AB

(b) (i) Find the equation of the line perpendicular to AB passing through $(2,5)$

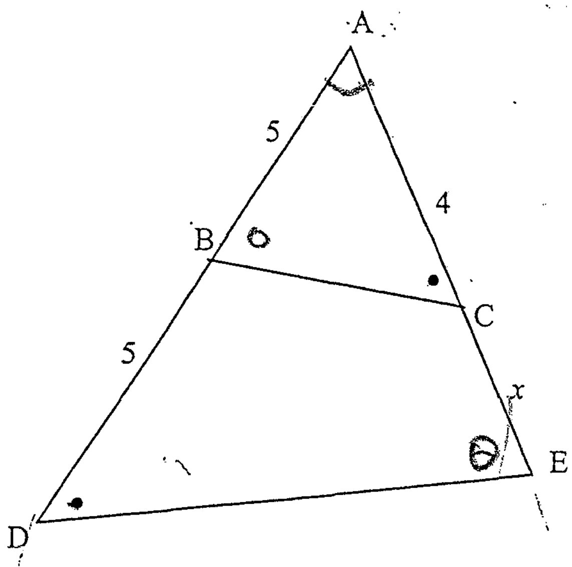
(ii) What angle does this line make with the x-axis?

(c) Graph the region on the number plane for which $y > 3$ and $x + y \leq 3$

$0 \leq 3$

Question 3.

(a)



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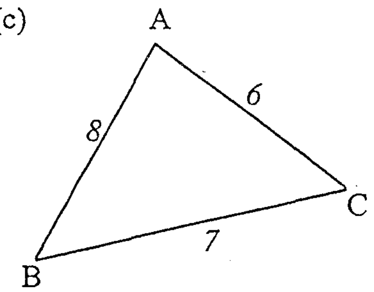
Find x

(b) Write the <u>exact</u> value of (i) $\cos 30^0$
(ii) $\tan 330^0$

(c)



Use the cos rule to find $\angle$ B correct to the nearest minute.

Question 4.

(a) Solve for x:

$$(x-2)(x+5) = 8$$

(b) Use the quadratic formula to find x correct to 1 decimal place:
$2x^2 - x - 14 = 0$

(c) (i) Graph on the number plane the piecemeal function

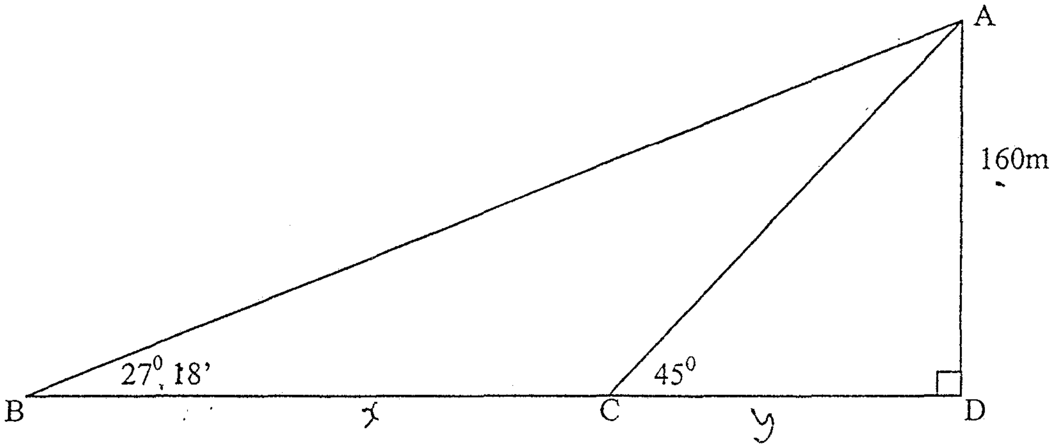
$$f(x) \begin{cases} |2x| & \text{for } x \leq -2 \\ 4 & \text{for } -2 < x < 2 \\ 2x & \text{for } x \geq 2 \end{cases}$$

(ii) State whether the function $y = f(x)$ is odd, even or neither

(iii) State whether the function $y = f(x)$ is continuous or discontinuous.

Question 5.

(a)



An observer in a boat rows towards a cliff which is 160m high.
At point B, the angle of elevation of the cliff top is 27^0 18'
At point C, the angle of elevation of the cliff top is 45^0

What is the distance BC to the nearest metre?

(b) The area enclosed by the curve $y = x^2$, the y-axis and the line $y = a$ is exactly 10 u^2.
Find a to 2 decimal places.

(c) What is the primitive of e^x ?

Question 6.

(a) Find $\dfrac{dy}{dx}$ for

(i) $y = \dfrac{x-2}{x+4}$

(ii) $y = e^{2x}$

(iii) $y = sin\ 5x$

(iv) $y = log_e\ (tan\ x)$

(b) Find the equation of the tangent to the curve $y = x^2 - x$ at the point (2, 2)

(c) Locate the stationary points on $y = x^3 - 3x$ and determine their nature.

Question 7.

(a) Seeds from a particular plant have 2 chances in 5 of surviving to flowering stage. $\frac{3}{4}$ of the surviving plants bear pink flowers and $\frac{1}{4}$ bear white flowers. Three seeds are planted. What is the probability:

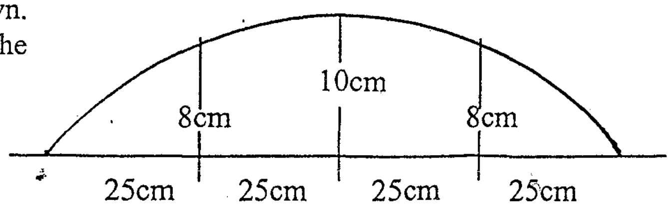
chl

(i) none survive
(ii) all 3 survive and bear pink flowers
(iii) at least one pink-flowering plant survives

(b) \$1000 is deposited in an account that pays 5%pa interest.

(i) How much will it be worth after 15 years?
(ii) If \$1000 is deposited at the beginning of each year into an account that pays 5% pa interest, what will be the balance at the end of 15 years.

(c) A speed bump has cross-section as shown. Use Simpson's Rule to find the area of the cross-section.
Hence find the volume of concrete required for a speed-bump for a road 10m wide.
Answer in cubic metres.



x10

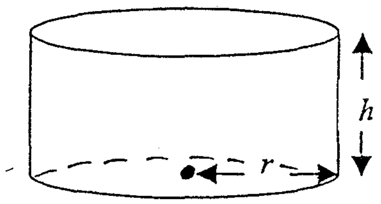
Question 8.

(a) Find the value(s) of k for which $x^2 - (k+2)x + (4k-4) = 0$

has
(i) two equal roots
(ii) one root the reciprocal of the other
(iii) one root equal to 8

(b) Solve for x: $4^x - 7.2^x - 8 = 0$

(c)



Given $V = \pi r^2 h$, and $r + h = 12cm$

Show that $\dfrac{dv}{dr} = 3\pi r(8-r)$

and hence show that the <u>maximum</u> volume of the cylinder is $256\pi\ cm^3$

Question 9.

(a) A curve of $y = f(x)$ passes through the point (2, 5) and (3, a).
If its gradient function is given by $f^{1}(x) = 3x^2 - 4x$
Find the value of a.

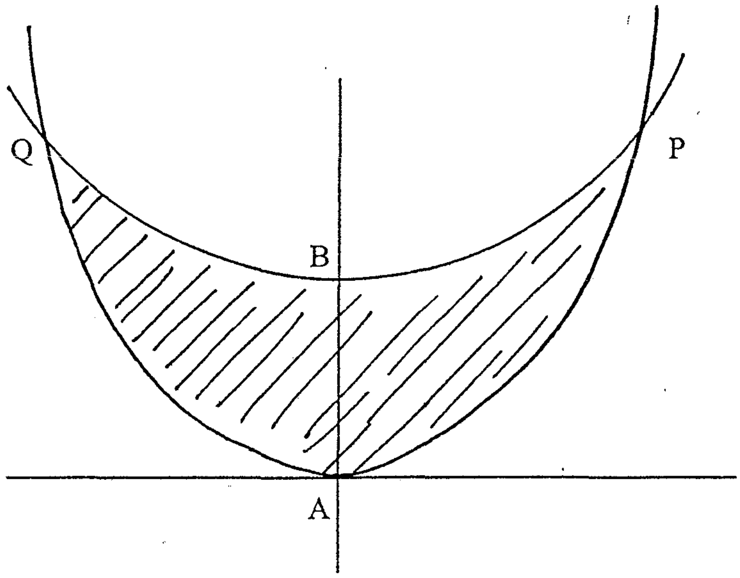
(b) A parabola has focus at (2, 4) and vertex at (2, -2)
Find (i) its focal length
(ii) the equation of the parabola
(iii) the equation of its axis of symmetry
(iv) the equation of its directrix

(c) (i) By completing the square, or otherwise, find the centre of the circle with equation $x^2 + 6x + y^2 - 2y = 15$

(ii) Find algebraically the points A and B where the circle cuts the y-axis

(iii) What is the area of ΔABC?

Question 10.



(a) The two curves $y = 2x^2$ and $y = x^2 + 1$ intersect at P and Q as shown.

(i) Find the coordinates of B, P and Q.

(ii) The shaded region is rotated about the y-axis to form a solid bowl shape. Find the volume of the bowl in terms of π

(b) Sketch the curve $y = 2sinx$ $\quad 0 \le x \le 2\pi$
Find the area enclosed by the curve and the x-axis

(c) The curve $y = 1 + \tan x$ between $x = 0$ and $x = \dfrac{\pi}{4}$ is rotated about the x-axis. Find the volume of the solid so formed.

-- **END OF PAPER** --

Hassan Salem

SYDNEY GIRLS HIGH SCHOOL - 2002 MATHEMATICS
TRIAL

Question 1

a) $2x-5=0$ $\quad x+2=0$

$2x=5$ $\quad x=-2$ ✓

$x=\frac{5}{2}$

$\therefore x=\frac{5}{2}, -2$

b) $\frac{e^4-1}{e^4+1} = \frac{53.59815}{55.59815} = 0.964$ (3 sig figs) ✓ ✓

c) $x+5=3$ $\quad x+5=-3$

$x=-2$ ✓, $x=-8$ ✓

d) $\frac{6x-5(x-1)}{10}$

$=\frac{6x-5x+5}{10}$ ✓

$=\frac{x+5}{10}$ ✓

12

e) let $x=0.\dot{3}\dot{1}$

~~$10x=3.\dot{1}\dot{3}$~~

$100x=31.\dot{3}\dot{1}$

$\therefore 99x=31$

$x=\frac{31}{99}$ ✓

Question 2

a) (i) $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad A(2,3)\ B(-2,-7)$

$=\sqrt{(-2-2)^2+(-7-3)^2}$

$=\sqrt{116}=\sqrt{4}\times\sqrt{29}$ ✓

$=2\sqrt{29}$ ✓

(ii) $\left(\frac{2+-2}{2}, \frac{3+-7}{2}\right) = (0,-2)$ ✓

(iii) $A(2,3)$ $B(-2,-7)$

$$m_{AB} = \frac{-7-3}{-2-2}$$

$$= \frac{-10}{-4} = \frac{10}{4} = \frac{5}{2} \checkmark$$

(iv) $y-3 = \frac{5}{2}(x-2)$

$$y-3 = \frac{5x-10}{2} \checkmark$$

$$2y-6 = 5x-10$$

$$5x-2y-10+6=0$$

$$5x-2y-4=0 \checkmark$$

12

(v) y intercept, when $x=0$

$$0-2y-4=0$$

$$-2y=4$$

$$y=-2$$

$\therefore$ y intercept is $(0,-2)$ ✓

b) (i) perpendicular to AB ie

$$m_{AB} \times m_{\perp AB} = -1$$

$$\therefore \frac{5}{2} \times m = -1$$

$$m = -1 \div \frac{5}{2}$$

$$= -\frac{2}{5} \checkmark$$

$$\therefore y-5 = -\frac{2}{5}(x-2)$$

$$y-5 = \frac{-2x+4}{5}$$

$$5y-25 = -2x+4 \checkmark$$

$$2x+5y-29=0$$

(ii) $5y = -2x+29$

$$y = \frac{-2}{5}x + \frac{29}{5}$$

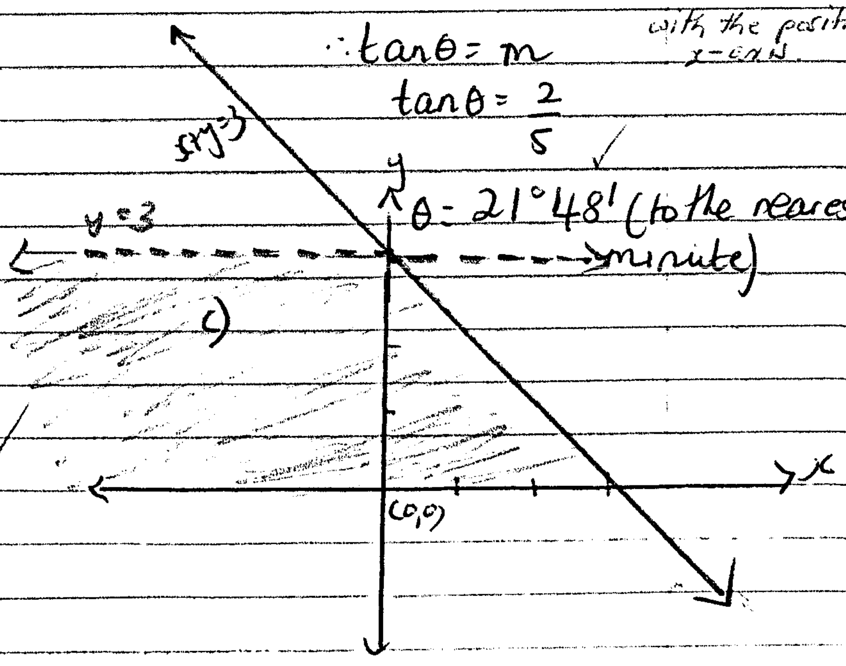
$$\therefore m = -\frac{2}{5}$$

$$\therefore \tan\theta = m$$

$$\tan\theta = \frac{2}{5} \checkmark$$

$\theta = 21°48'$ (to the nearest minute)

$\theta = 158°12'$ with the positive x-axis.



Question 3

a) In Δ's ABC and ADE

∠A is common

∠ACB = ∠~~ADE~~ AED (given)

∴ ΔABC ||| Δ~~ADE~~ AED (equiangular)

$\therefore \frac{AD}{AB} = \frac{AE}{AE}$ (corresponding sides in similar Δ's)

AC AB

$\therefore \frac{10}{5} = \frac{x+4}{4}$ $\quad \frac{10}{4} = \frac{4+x}{5}$

10

$40 = 5x + 20$ $\quad 50 = 16 + 4x$

$5x = 20$ $\quad 34 = 4x$

$x = 4$ units $\quad \frac{34}{4} = x \Rightarrow x = 8\frac{1}{2}$

b) (i) $\cos 30° = \frac{\sqrt{3}}{2}$ ✓

(ii) $\tan 330° = -\frac{1}{\sqrt{3}}$ ✓

c) $\cos\angle B = \frac{8^2 + 7^2 - 6^2}{2\times8\times7}$

$\cos\angle B = \frac{77}{112}$ ✓

$\cos\angle B = 0.6875$ ✓

$\therefore \angle B = 46°34'$ (to the nearest minute)

Question 4

a) $x^2 + 5x - 2x - 10 = 8$

$x^2 + 3x - 10 = 8$

$x^2 + 3x - 10 - 8 = 0$ ✓

$x^2 + 3x - 18 = 0$ ✓

$(x-3)(x+6) = 0$

$x = 3,\ x = -6$ ✓

b) $2x^2 - x - 14 = 0$

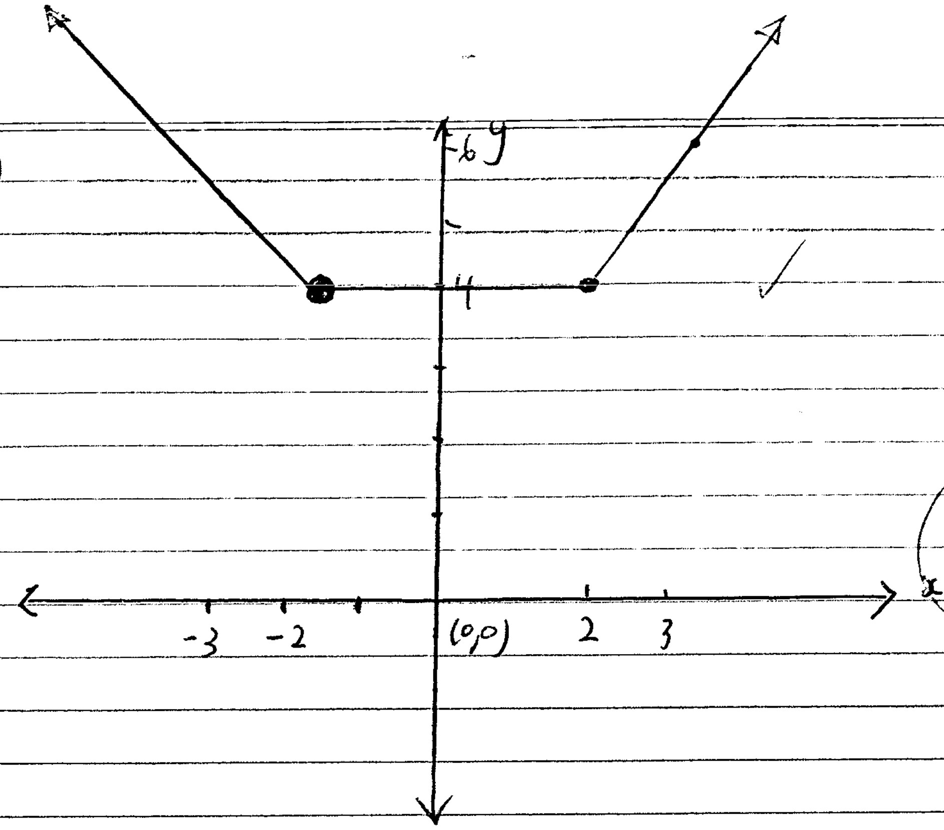
$x = \frac{1 \pm \sqrt{1 - 4\times2\times-14}}{4}$

$x = \frac{1 \pm \sqrt{113}}{4}$ ✓

✓

$\therefore x = 2.9,\ x = -2.4$ (to 1 d.p)

c) (i)



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(ii) even function

(iii) ~~discontinuous~~ continuous

Question 5

a) $\angle ACB = 135°$ (angle sum of straight line is 180°)
$\therefore \angle BAC = 17°42'$ (angle sum of triangle = 180°)
$\angle CAD = 45°$ (angle sum of Δ = 180°)
$\therefore \Delta ADC$ is isosceles
$\therefore CD = 160m$

$\sin 45° = \frac{160}{AC}$

$AC = \frac{160}{\frac{1}{\sqrt{2}}}$

$160 \div \frac{1}{\sqrt{2}}$

$160 \times \frac{\sqrt{2}}{1}$

$AC = 160\sqrt{2}$ metres

In ΔABC

$\frac{BC}{\sin 17°42'} = \frac{AC}{\sin 27°18'}$ (sine rule)

Quicker to find BD

using $\tan 27°18' = \frac{160}{BC+160}$

$\therefore BC + 160 = \frac{160}{\tan 27°18'}$

$BC = \frac{AC \sin 17°42'}{\sin 27°18'} \Rightarrow BC = 149.9$
$= 150m$

$AC = 160\sqrt{2}$

$\therefore BC = \frac{160\sqrt{2} \times \sin 17°42'}{\sin 27°18'}$

$\therefore BC = 149.99\ldots$
$= 150m$ (to the nearest metre)

b) $y = x^2$

$x^2 = y$

$x = \pm\sqrt{y}$

$\therefore \int_0^a \cancel{\pm}\, y^{1/2}\, dy = \left[\cancel{\pm} \dfrac{y^{3/2}}{3/2} \right]_0^a = 10$ ✓

$\sqrt{y^3} \div \dfrac{3}{2}$

$\sqrt{y^3} \times \dfrac{2}{3}$

$\left[\dfrac{\pm 2\sqrt{y^3}}{3} \right]_0^a = 10$

12

$\pm \dfrac{2\sqrt{a^3}}{3} - \dfrac{2\sqrt{0}}{3} = 10$

$\cancel{\pm} \dfrac{2\sqrt{a^3}}{3} = 10$ ✓

$\therefore \dfrac{2\sqrt{a^3}}{3} = 10$

$2\sqrt{a^3} = 30$

$\sqrt{a^3} = 15$

$a^3 = 225$ ✓

$a = 6.08$ (to 2 d.p)

$\therefore a = 6.08$

~~$-\dfrac{2\sqrt{a^3}}{3} = 10$~~

~~$-2\sqrt{a^3} = 30$~~

~~$\sqrt{a^3} = -15$~~

~~$a^3 = 225$~~

↑ same solution

c) $\int e^x dx = e^x + c$ ✓

Question 6

a) (1) $u = x - 2 \quad v = x + 4$

$u' = 1 \quad v' = 1$

$\therefore \dfrac{dy}{dx} = \dfrac{x + 4 - (x - 2)}{(x+4)^2} = \dfrac{\cancel{x} + 4 \cancel{- x} + 2}{(x+4)^2} = \dfrac{6}{(x+4)^2}$ ✓

(ii) $y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x}$ ✓

(iii) $y = \sin 5x$

$\frac{dy}{dx} = \cos 5x \times 5$

$= 5\cos 5x$ ✓

(iv) $\frac{dy}{dx} = \frac{\sec^2 x}{\tan x}$ ✓

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b) $y = x^2 - x$

$\frac{dy}{dx} = 2x - 1$ ✓

at $x = 2$, $m_T = 2 \times 2 - 1$

$= 3$ ✓

$\therefore y - 2 = 3(x-2)$

$y - 2 = 3x - 6$ ✓

$3x - y - 4 = 0$

c) $y = x^3 - 3x$

$\frac{dy}{dx} = 3x^2 - 3$

to find stationary points, let $\frac{dy}{dx} = 0$

$\therefore 3x^2 - 3 = 0$

$\cancel{3x}\ 3(x^2 - 1) = 0$

$3(x+1)(x-1) = 0$ ✓

$\therefore x = \pm 1$

when $x = 1$, $y = -2$

when $x = -1$, $y = 2$ ✓

$\therefore$ stationary points are $(1, -2)$ and $(-1, 2)$

$y'' = 6x$ ✓

at $x = 1$, $y'' = 6 > 0$ $\therefore$ $(1, -2)$ is a minimum turning point

at $x = -1$, $y'' = -6$, < 0 $\therefore$ $(-1, 2)$ is a maximum turning point ✓

b) (i) $A = P\left(1+\frac{R}{100}\right)^n$

$A = 1000\left(1+\frac{5}{100}\right)^{15}$

$A = 1000(1.05)^{15}$ ✓

$= \$2078.93$ (to 2 dp)

9

(ii) $A_2 = 1000(1.05)^1$

$+$

$1000(1.05)^2$

$A_2 = 1000(1.05) + 100(1.05)^2$

$= 1000(1.05 + 1.05^2)$

$\therefore A_{15} = 1000(\underbrace{1.05 + 1.05^2 + 1.05^3 + \dots}_{G.P} 1.05^{15})$ ✓

with $a = 1.05$, $r = 1.05$, $n = 15$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_n = \frac{1.05(1.05^{15} - 1)}{0.05}$

$\therefore A_{15} = 1000\left(\frac{1.05(1.05^{15}-1)}{0.05}\right)$ ✓

$\$22657.49$ (to 2 dp) ✓

c)

x	0	25	50	75	100
y	0	8	10	8	0
	1st	4	2	4	Last

$h = 25$

$A \doteqdot \frac{25}{3}\left[0 + 0 + 4(8+8) + 2(10)\right]$ ✓

Area $\doteqdot 700 cm^2$

$\therefore$ Volume $= 700 \times 10$

$= 7000 cm^3$ ✓

Question 7

a) (i) $P(\text{surviving}) = \frac{2}{5}$

$P(\text{not surviving}) = 1 - \frac{2}{5}$

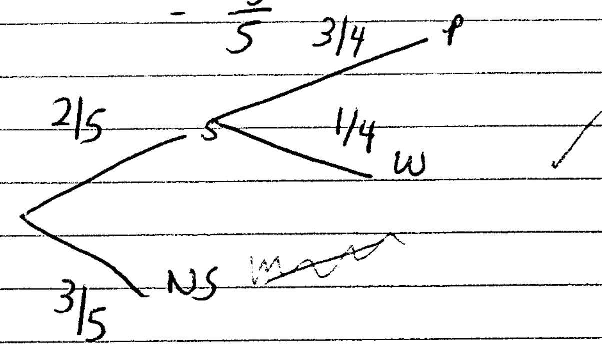
$= \frac{3}{5}$

(i) $P(\tilde{S}.\tilde{S}.\tilde{S})$

$= \left(\frac{3}{5}\right)^3$

$= \frac{27}{125}$

(ii)



$P(SP) = \frac{2}{5} \times 3/4 = \frac{3}{10}$

three ~~flower~~ plants $= \frac{3}{10} \cancel{\times 3}$ $\quad \left(\frac{3}{10}\right)^3$

$= \frac{9}{10}$ $\quad = \frac{27}{1000}$

(iii) ~~$1 - P(\text{non-surviving})$~~

$1 - 1$

$1 - P(\text{None surviving})$

$= 1 - \frac{27}{125}$

$= \frac{98}{125}$

Question 8

a) (i) $\Delta = (k+2)^2 - 4\times1\times(4k-4)$

$k^2+4k+4-4(4k-4)$

$k^2+4k+4-16k+16$

$\Delta = k^2-12k+20$

equal roots ie $\Delta=0$

$\therefore k^2-12k+20=0$

$(k-2)(k-10)=0$

$\therefore k=2 \quad k=10$

(ii) let roots be α and $\frac{1}{\alpha}$

$\alpha\times\frac{1}{\alpha}=1$

$\therefore \frac{4k-4}{1}=1$

$4k-4=1$

$4k=5$

$k=\frac{5}{4}$

(iii) subing $x=8$

$64-(k+2)\times8+(4k-4)=0$

$64-8(k+2)+4k-4=0$

$\cancel{64}-8k-\cancel{16}+4k-\cancel{4}=0$

$44-4k=0$

$4k=44$

$k=11$

b) let $u=2^x$

$\therefore u^2-7u-8=0$

$(u+1)(u-8)=0$

$\therefore u=-1, \quad u=8$

$\therefore 2^x=-1 \qquad 2^x=8$

~~$\ln 2^x = \ln -1$~~ $\qquad 2^x=2^3$

$\therefore$ no solutions $\qquad \therefore x=3$

$\therefore$ the only solution is $x=3$

c) $h=12-r$

$V=\pi r^2h$

$=\pi r^2(12-r)$

$12\pi r^2-\pi r^3$

$\therefore \frac{dV}{dr}=24\pi r-3\pi r^2$

$3\pi r(8-r)$

to find stationary points, let $\frac{dV}{dr}=0$

$\therefore 3\pi r(8-r)=0$

$3\pi r=0, \quad 8-r=0$

$r=0 \qquad r=8$

$r\neq0$ as it is a length

$\therefore r=8\text{cm}$

~~$u=3\pi r \quad v=8-r$~~

~~$u'=3\pi \quad v'=-1$~~

~~$\frac{d^2V}{dr^2}=3\pi(8-r)+3\pi r$~~

~~at $r=8$, $V''=-75.39\ldots$~~ <0

$\frac{dV}{dr}=24\pi r-3\pi r^2$

$\frac{d^2V}{dr^2}=24\pi-6\pi r$

at $r=8$, $\frac{d^2V}{dr^2}=-75.39\ldots<0$ $\therefore$ at $r=8$ it is a maximum.

$h+r=12 \qquad h=4$

$h+8=12$

$V=\pi r^2h$

12

$\therefore V = \pi r^2 h$

$= \pi \times 8 \times 8 \times 4$

$= 256\pi \text{ cm}^3$ ✓

Question 9

$\int 3x^2 - 4x \, dx = \frac{3x^3}{3} - \frac{4x^2}{2} + c$

$f(x) = x^3 - 2x^2 + c$

passes through $(2,5)$ ✓

$\therefore 5 = c + 0$

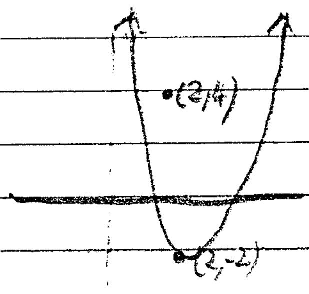
$\therefore c = 5$

$\therefore f(x) = x^3 - 2x^2 + 5$ ✓

passes through $(3, a)$

$\therefore a = 14$ ✓

b) (i)



focal length = 6 units ✓

(ii) ~~...~~

$(x-2)^2 = 24(y+2)$ ✓

$4a = 24$

$a =$

(iii) $x = 2$ ✓

(iv) $y = -8$ ✓

c) (i) $x^2 + 6x + 9 + y^2 - 2y + 1 = 15 + 1 + 9$

$(x+3)^2 + (y-1)^2 = 25$

$\therefore$ centre $= (-3, 1)$ ✓

(ii) cuts y axis when $x = 0$

$\therefore (0+3)^2 + (y^2 - 2y + 1) = 25$

* Try this

$3^2 + (y-1)^2 = 25$

$(y-1)^2 = 16 \Rightarrow y - 1 = \pm 4$

$y = 5$ or -3

* Too long

$9 + y^2 - 2y + 1 = 25$

$10 + y^2 - 2y = 25$

$y^2 - 2y + 10 - 25 = 0$

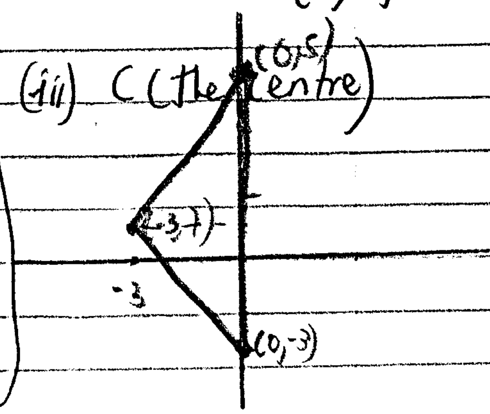
$y^2 - 2y - 15 = 0$

$(y+3)(y-5) = 0$

$y = -3, \; y = 5$

$\therefore$ A and B are $(0,-3)$ and $(0,5)$ ✓

(iii) C (the centre)



AB = 8 units

height = 3

$\therefore \frac{1}{2} \times 3 \times 8$

$= \frac{24}{2}$

$= 12 \text{ units}^2$

12

Question 10

a) (i) intersection: $2x^2 = x^2 + 1$

$x^2 = 1$

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$\therefore x = \pm 1$

when $x = 1$, $y = 2$

when $x = -1$, $y = 2$

$\therefore Q(-1, 2)$ and $P(1, 2)$

$\cancel{B}$ $y = x^2 + 1$

at B, $x = 0$

$\therefore y = 1$

$\therefore B(0, 1)$

$\therefore B(0,1)$; $P(1,2)$ and $Q(-1,2)$

(ii) $y = 2x^2$ $\quad\quad$ $y = x^2 + 1$

$2x^2 = y$ $\quad\quad$ $y - 1 = x^2$

$x^2 = \frac{y}{2}$ $\quad\quad$ $x^2 = y - 1$

$\cancel{x = \frac{\sqrt{y}}{\sqrt{2}}}$

$\frac{y}{2} - y + 1$

$\frac{y - 2(y+1)}{2}$

$\frac{y - 2y - 2}{2}$

$\therefore V = \pi \int_0^1 \frac{y}{2} - (y-1)\, dy$

$V = \pi \int_0^1 \frac{y - 2y + 2}{2}\, dy$

$\frac{\pi}{2} \int_0^1 y - 2y + 2\, dy = \frac{y^2}{2} - \frac{2y^2}{2} \cancel{1} + 2y$

$\frac{\pi}{2}\left[\frac{y^2}{2} - y^2 + 2y\right]_0^1$

$\left[\left(\frac{1}{2} - 1 + 2\right)\right] - 0$

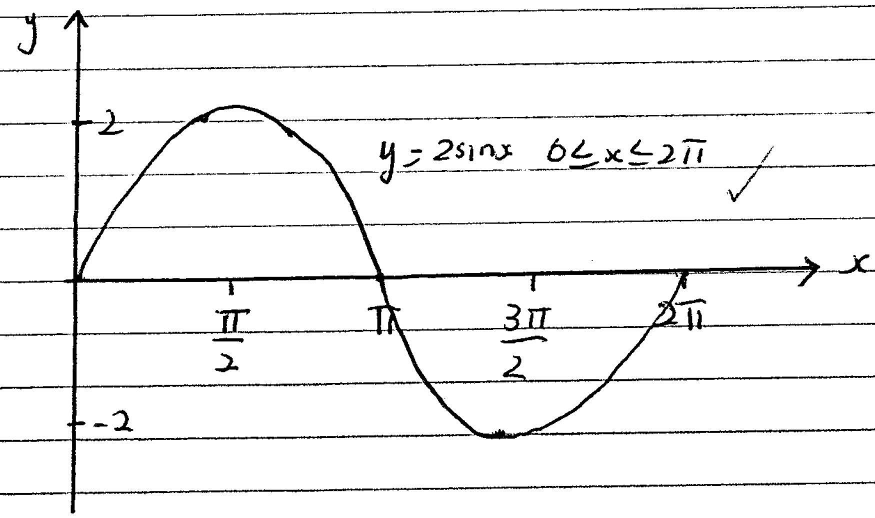
$= \frac{5}{2}$ $\quad \frac{3}{2}\times\frac{\pi}{2}$

$\frac{5}{2}\times\frac{\pi}{2} = \frac{3\cancel{5}\pi}{4}$ units^3

$\cancel{\text{Total volume} = 2\times\frac{5\pi}{4}}$

$\cancel{= \frac{10\pi}{4} = \frac{5\pi}{2} \text{ units}^3}$

b)



Area $= 2\times\int_0^{\pi} 2\sin x\,dx = \left[-2\cos x\right]_0^{\pi}\times 2$ ✓

$\left[2 - -2\right] = 4$

$4\times 2 = 8$ units^2 ✓

c) $\pi\int_0^{\frac{\pi}{4}}(1+\tan x)^2\,dx = \cancel{(1+\tan x)(1+\tan x)}$

$= \pi\int_0^{\pi/4} 1+2\tan x+\tan^2 x\,dx$ ✓

$\sin^2\theta+\cos^2\theta=1$
$\tan^2\theta+1=\sec^2\theta$
$\tan^2\theta=\sec^2\theta-1$ ✓

$= \pi\int_0^{\pi/4} 1+2\tan x+\sec^2 x-1\,dx$

$$\pi \int_0^{\pi/4} 1 + 2\left(\frac{\sin x}{\cos x}\right) + \sec^2 x - 1 \, dx = \pi\left[x + -2\ln(\cos x) + \tan x - x\right]$$

$$\pi \left[\cancel{x} - 2\ln(\cos x) + \tan x - \cancel{x}\right]_0^{\frac{\pi}{4}}$$

$$\pi\left[-2\ln(\cos x) + \tan x\right]_0^{\pi/4}$$

$$\pi\left[-2\ln\left(\frac{1}{\sqrt{2}}\right) + 1\right] - [0]$$

$$\cancel{= 2\pi\left(\ln\left(\frac{1}{\sqrt{2}}\right) + 1\right) \text{ units}^3}$$

$$\pi\left[-2(\ln 1 - \ln\sqrt{2}) + 1\right]$$

$$= \pi\left[2\ln\sqrt{2} + 1\right]$$

$$= \pi(\ln 2 + 1) \text{ units}^3$$

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